THERMIONIC SCREENING OF BODIES IN ATMOSPHERE AND INTERPLANETARY SPACE

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ABSTRACT

This paper considers problems which accompany thermionic emission of electrons from a hot body surrounded by a plasma. In the absence of other mechanisms, an electric potential is established at the surface of the body through the balance of thermionic emission and accretion of electrons from the external plasma. Analytical solutions are obtained for the electric potential field and the electron density distribution around the body. A possible application of this analysis to objects in space is indicated.

I. INTRODUCTION

An object in space may become hot while approaching a hot stellar body like the sun, or while entering a dense atmosphere like that of the earth. Long before such metallic objects melt, evaporate, or ablate, they may acquire temperatures which are sufficient to cause a copious emission of electrons from the surface. Therefore, the temperatures lower than, and in the neighborhood of, the melting point are of interest to us in this paper. As a matter of convenience and without serious loss of generality, we will regard iron as a reference substance composing the objects in space, and hence consider temperatures lower than $1600^{\circ}K$. The analytical formulae are, however, applicable to any other specific case of a surface capable of thermionic emission.

Thermionic emission is very sensitive to temperatures; the emitted electron flux is of the order of 10^{12} and 10^{18} electrons/cm^2-sec at surface temperatures of $1000^{\circ}K$ and $1600^{\circ}K$ respectively, from a material of work function $W_o = 3$ ev. The emission of electrons from the object's surface leaves a positive surface charge. A great majority of the emitted electrons describe ballistic orbits and return to the surface, while a certain number of those in the high energy tail of the energy spectrum are able to escape from the potential field of the object. The positive charge at the object's surface is established by these escaping electrons, and the rate of escape of the emitted electrons decreases with an increase in the surface potential. Furthermore, if the object is surrounded by a plasma, the plasma accretion alone has a tendency to impart a negative charge to the object's surface. Therefore, a steady potential can be established at the object's surface when the net negative charge leaving the object due to thermionic emission is completely replenished by the net negative charge brought to the surface by the accretion from the surrounding plasma. The magnitude of the equilibrium surface potential is then determined from the balance of the plasma accretion current and the escape component of the thermionic emission current.

There are other mechanisms (Chopra 1961) in which an object may acquire an electric charge. An effect of considerable interest is connected with the photoelectric emission and accretion of electrons. The photoelectric effect is important for objects on the day side of the earth and for surfaces exposed to the sun. In certain cases, it is comparable to, and at times may even become more significant than, the thermionic emission. We will, however, limit the analysis of the present paper to only thermionic emission and leave these other considerations for a subsequent paper.

The incoming plasma electron flux and the thermionic electrons constitute a plasma cloud with most of the contribution to the electron density in the cloud coming from the ballistic component of the emitted electrons. This plasma cloud screens the electric potential on the body. An analytical expression for the density distribution in terms of potential φ_o and work function W_o is obtained by solving the equations of Poisson and the conservation of energy and momentum. This analytical expression is substituted back in the Poisson equation which is then solved numerically to yield potential distribution as a function of distance from the surface.

It may be mentioned here that the problem considered in this paper bears a certain analogy to the problem of the exosphere. In the exosphere problem, the particles are projected outwards corresponding to the temperature of the base layer. One of us (C.S. Shen) has successfully applied (Shen 1963) the present analysis (after some modifications) to the structure of the planetary exosphere, and has obtained an analytical expression for the density distribution.

II. FORMULATION OF THE PROBLEM - BASIC EQUATIONS

Let us consider a spherical object with an equilibrium surface potential φ_o and surface temperature T, surrounded by screening charges due to thermionic emission and a rarefied external plasma with electron density n_e and ion density n_e/Z (where Ze is the ionic charge at a temperature T_p). When the thermionic emission is stronger than the plasma accretion and the object is moving slower than the mean thermal speed of the plasma electrons ($\sim 10^7$ cm/sec) the potential $\varphi(r)$ and the screening electron density $\rho(r)$ are, to a first degree of approximation, spherically symmetrical, and are given by

$$\nabla^2 \varphi(r) = -\frac{e}{\epsilon_o} \rho(r) \tag{1}$$

where $e = 4.8 \times 10^{-10}$, e.s.u. is the electron charge, $\epsilon_o = 1$ is the permitivity of the medium, and r is the radial distance measured from the center of the spherical body.

The screening electron density $\rho(r)$ consists of three parts

$$\rho(r) = \rho_b(r) + \rho_{esc}(r) + \rho_p(r) \quad . \tag{2}$$

Here $\rho_b(r)$ is the ballistic component which is due to the electrons emitted from the surface with velocities less than the escape velocity; these particles describe ballistic orbits in the electric potential field of the body and return to the surface. The escape component $\rho_{esc}(r)$ is due to the electrons emitted with velocities exceeding the escape velocity; these particles do not return to the charged body. The third component $\rho(r)$ is due to the accretion from the surrounding plasma. Among these $\rho_b(r)$ contributes about 90 percent to the local electron density (as can be seen from later calculations). Also, in the steady state condition, the escape component of the thermionic electron flux is equal to the incoming plasma accretion flux. Therefore, to simplify one of our later calculations, we can set $\rho_{esc}(r) = \rho_p(r)$.

Assuming that the electrons inside the metallic body have velocities given by the Fermi distribution law, the number of electrons having velocities in the range $(\vec{v}, \vec{v} + d\vec{v})$ and hitting a unit area of the surface (inside) is given by

$$J(\vec{v}) = \frac{4\pi m^3}{h^3} \frac{v_r v_t dv_r dv_t}{e^{(E-E_f)/kT}+1} \tag{3}$$

where

$$E = 1/2m(v_r^2 + v_t^2)$$

and $m = 9 \times 10^{-28}$g is the electron mass, $h = 6.27 \times 10^{-27}$erg/sec is the Planck's constant, E_f is the Fermi energy, and v_r and v_t are the components of the velocity $\vec{v}$ in directions parallel and transverse to the radius vector $\vec{r}$.

If we denote the velocity of the electron at the position

$\vec{r}$, $(r>R)$, by $\vec{u}(r)$, then the principles of conservation of energy and angular momentum require that

$$ru_t = Rv_t \tag{4}$$

and

$$1/2m(u_r^2+u_t^2)-e\varphi(r) = 1/2m(v_r^2+v_t^2)-W_o-E_f-e\varphi_o(r) \tag{5}$$

where R is the radius of the body.

Equations (4) and (5) yield

$$u_r^2 = v_r^2+(1 - \frac{R^2}{r^2})v_t^2 - \frac{2}{m}\{e(\varphi_o-\varphi) + E_f + W_o\} \tag{6}$$

which provides a stringent condition for an electron emitted from the surface to reach the radial distance r. Only those electrons with initial velocity $\vec{v}$ satisfying the inequality

$$v_r^2 + (1 - \frac{R^2}{r^2})v_t^2 - \frac{2}{m}\{e(\varphi_o - \varphi) + E_f + W_o\} \geq 0 \tag{7}$$

can reach position r. These electrons may be divided into two categories:

1) Ballistic Component: These electrons satisfy Equation (7) and have velocities less than the velocity of escape such that

$$1/2mv^2 - W_o - E_f < e\varphi_o \tag{8}$$

and hence describe ballistic orbits.

2) Escape Component: These electrons satisfy Equation (7) and have velocities equal to or exceeding the velocity of escape such that

$$1/2mv^2 - W_o - E_f \geq e\varphi_o \tag{9}$$

and describe escape trajectories.

These classifications are important in the evaluation of electron density and may be illustrated diagrammatically as in Figure 1. Curves I, II, and III describe equations

$$v_r^2 + v_t^2 = (\tfrac{2}{m}) [E_f + W_o + e\varphi_o] \tag{10}$$

$$v_r^2 + (1 - \alpha^2)v_t^2 = (\tfrac{2}{m}) [E_f + W_o + e\{\varphi_o - \varphi\}] \quad , \tag{11}$$

and

$$v_r^2 = (\tfrac{2}{m}) [E_f + W_o] \quad , \tag{12}$$

where $\alpha = R/r$. These curves represent a circle, an ellipse and a straight line in the same order and distribute the thermionic electrons in various velocity domains.

The electrons with velocity domains external to the circle and the straight line - region A - are the escape electrons which do not return to the body. The electrons with velocities in the domain enclosed by the circle and the ellipse - region B - belong to the ballistic group with more than the necessary radial component of the velocity to reach the position r. These electrons are counted twice in calculating the electron density distribution and make a dominant contribution to the local electron population. The electrons corresponding to region C - enclosed by the ellipse and the straight line - also belong to the ballistic group but do not possess enough radial velocity to reach position r. Therefore, these particles do not contribute to the local electron density. The straight line represents the least value of the radial velocity that an electron must acquire before it can surmount the surface barrier. Therefore, the electrons corresponding to region D in Figure 1 are not able to get out of the surface of the metallic body.

III. ELECTRON DENSITY AS A FUNCTION OF POTENTIAL

The contribution of the thermionic electrons with the initial (just inside the surface) velocities in the range $(\vec{v}, \vec{v} + d\vec{v})$ to the electron population in a shell of radii r and r + dr is determined by the product of the corresponding electron flux $J(\vec{v})d\vec{v}$ and the time $dt = dr/u_r$ spent by these electrons in traversing the thickness dr of the shell. This contribution $d\rho_{th}(r)$ is given by

$$d\rho_{th}(r) = \alpha^2 \int_{u_r>0} \frac{J(\vec{v})d\vec{v}}{u_r} \tag{13}$$

which with the help of Equations (3) and (6) yields the expression for the thermionic component $\rho_{th}(r)$ of the electron density at position r,

$$\rho_{th}(r) = 4\pi \left(\frac{m}{h}\right)^3 \alpha^2 \; [2I_1 + I_2] \quad , \tag{14}$$

where

$$I_1 = \iint_{\text{Region B}} \frac{v_r v_t dv_r dv_t}{[v_r^2+(1-\alpha^2)v_t^2-(2/m)\{E_f+W_o+e(\varphi_o-\varphi)\}]^{1/2}[e^{\{m(v_r^2+v_t^2)-2E_f\}/2kT}+1]} \tag{15}$$

and

$$I_2 = \iint_{\text{Region A}} \frac{v_r v_t dv_r dv_t}{[v_r^2+(1-\alpha^2)v_t^2-(2/m)\{E_f+W_o+e(\varphi_o-\varphi)\}]^{1/2}[e^{\{m(v_r^2+v_t^2)-2E_f\}/2kT}+1]} \tag{16}$$

and the limits of the integrals I_1 and I_2 are set in accordance with Equations (10) - (12) and Figure 1, and the weight factors are inserted as explained in the preceding section. Including the contribution of the external plasma, the total electron density $\rho(r)$

at the position r becomes

$$\rho(r) = \rho_{th}(r) + \rho_p(r) = 8\pi(m/h)^3\alpha^2(I_1 + I_2) \ . \tag{17}$$

On introducing the following dimensionless parameters

$$X^2 = mv_r{}^2/2kT \ ; \quad Y^2 = mv_t{}^2/2kT \ ; \ \epsilon = E_f/kT \tag{18}$$

and

$$a = [E_f + W_o + e(\varphi_o - \varphi)]/kT \quad ,$$

Equation (17) reduces to

$$\rho(r) = \frac{8\pi(2mkT)^{3/2}\alpha^2}{h^3}\int\int\frac{XYdXdY}{[X^2+(1-\alpha^2)Y^2-a]^{1/2}[e^{X^2+Y^2-\epsilon}+1]} \tag{19}$$

$$X^2+(1-\alpha^2)Y^2-a\geq 0 \quad .$$

But,

$$X^2 + Y^2 - \epsilon \geq a - \epsilon \geq W_o/kT >> 1$$

and, therefore, we can neglect the unity term in comparison with the exponential term in the denominator of Equation (19). Furthermore, on setting

$$X^2 + (1-\alpha^2)Y^2 - \epsilon - (W_o/kT) = Z^2 \tag{20}$$

and

$$(1 - \alpha^2)^{1/2}(Y/X) = \tan \beta \tag{21}$$

in the last equation, and after some simplification, the expression for the total electron density $\rho(r)$ reduces to

$$\rho(r) = 2(2\pi mkT/h^2)^{3/2} e^{-\{W_o + e(\varphi_o - \varphi)\}/kT}$$

$$\left[1 - \{1-(R/r)^2\}^{1/2} e^{-R^2 e\ (\varphi_o - \varphi)/kT(r^2-R^2)} \right] \tag{22}$$

which is expressed as a function of the potential $\varphi(r) = \varphi$.

IV. REDUCED POISSON EQUATION

Substituting for the electron density $\rho(r)$ from Equation (22) in the Poisson Equation (1) and introducing the dimensionless quantities

$$\psi = e\varphi/kT$$

and (23)

$$\theta = W_o/kT \quad ,$$

we obtain

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = A(kT)^{1/2} e^{-(\psi_o - \psi)} \left[1-\left\{1-(R/r)^2\right\}^{1/2} e^{-(\psi_o-\psi)R^2/(r^2-R^2)}\right] \tag{24}$$

where

$$A = -2(e^2/\varepsilon_o) \ (2\pi m/h^2)^{3/2} \ e^{-\theta} \tag{25}$$

and the boundary conditions of the problem are

$$\psi(R) = \psi_o \tag{26}$$

at $r = R$, and

$$\psi(\infty) = 0 \tag{27}$$

at $r = \infty$.

In some cases of interest to us, we will find that the equilibrium potential energy $e\varphi_o$ is much greater than the thermal energy kT corresponding to the surface temperature T. This would then enable us to neglect, to a first approximation, the second term in Equation (24), and hence we have

$$\nabla^2 \ \psi(r) = Be^{-X} \tag{28}$$

with

$$B = A(kT)^{1/2}, \text{ and } X = \psi_o - \psi \ . \tag{29}$$

Equation (27) is identical to the so-called isothermal equation which has been solved for various boundary conditions and applied extensively to the problems pertaining to stellar structure by Chandrasekhar (1939).

V. DETERMINATION OF SURFACE POTENTIAL

The equilibrium value of the surface potential is determined from the balance of the escape component of the thermionic emission current and the plasma accretion current. The plasma accretion current consists of the electron and ion components. In the absence of a surface potential φ_o, the ion accretion current is smaller than the electron accretion current by a factor of the order of $(m_e/m_i)^{1/2}$. Therefore, only the relative initial magnitudes of the thermionic escape current and the plasma electron accretion current need be considered, and the ion accretion current may be neglected. Then, the surface potential φ_o is positive if the initial thermionic escape current is greater than the initial electron accretion current. It may, however, become necessary to include ion accretion in consideration of the magnitude of the surface potential φ_o, if the latter is negative.

Let us first consider the case of a positive surface potential. Then, the thermionic escape current is given by

$$J_{esc} = 8\pi R^2 (m/h)^3 \iint_{\text{Region A}} \frac{v_r v_t dv_r dv_t}{e^{-\{m(v_r^2 + v_t^2) - 2E_f\}/2kT} + 1} \quad (30)$$

As mentioned in Section II, the electrons in region A of Figure 1 must have radial and transverse velocity components such that

$$v_r^{\,2} > (2/m)\ (E_f + W_o)$$

and

$$v_r^{\,2} + v_t^{\,2} > (2/m)\ (E_f + W_o + e\varphi_o)\ .$$

Therefore, the expression for the thermionic escape component may be rewritten as

$$J_{esc} = 8\pi R^2 (m/h)^3 \Bigg[\int_{\sqrt{(2/m)\,(E_f + W_o)}}^{\sqrt{(2/m)\,(E_f + W_o + e\varphi_o)}} v_r dv_r \int_{\sqrt{(2/m)\,(E_f + W_o + e\varphi_o)}}^{\infty} \frac{v_t dv_t}{e^{-\{m(v_r^{\,2} + v_t^{\,2}) - 2E_f\}/2kT} + 1}$$

$$+ \int_{\sqrt{(2/m)\,(E_f + W_o + e\varphi_o)}}^{\infty} v_r dv_r \int_0^{\infty} \frac{v_t dv_t}{e^{-\{m(v_r^{\,2} + v_t^{\,2}) - 2E_f\}/2kT} + 1} \Bigg] \qquad (31)$$

Once again we can neglect the unity terms in comparison with the exponential terms in the denominators of the integrands in Equation (31). After carrying out the integration and making some simplifications, Equation (31) reduces to

$$J_{esc} = (4\pi RkT)^2 (m/h^3)(1+\psi) e^{-(\psi_o+\theta)} \quad . \tag{32}$$

For bodies moving slower than the mean thermal speed of the plasma electrons, the electron accretion is symmetrical about the body, and is given by

$$J_e = \frac{2\pi}{3} \eta_e n_e v_e R^2 e^{(T/T_p)\psi_o} \tag{33}$$

when n_e and v_e are the number density and the mean thermal speed of the plasma electrons, and η_e is the sticking coefficient defined as the fraction of the incident electrons transferring their charge to the body. In estimating the plasma accretion current J_p we further note that the ion accretion is further reduced by a factor $e^{-(T/T_p)\psi_o}$ and becomes negligibly small as compared to the electron accretion. Hence,

$$J_p \cong J_e = \frac{2\pi}{3} \eta_e n_e v_e R^2 e^{(T/T_p)\psi_o} \tag{34}$$

which, when combined with Equation (32) in the condition of equilibrium,

$$J_{esc} = J_p \quad , \tag{35}$$

yields

$$\frac{e^{[1 + T/T_p]\psi_o}}{1 + \psi_o} = \frac{24\pi m_e (kT)^2 e^{-\theta}}{\eta_e n_e v_e h^3} = 6 \times 10^{29} \frac{T^2 e^{-\theta}}{\eta n_e v_e} \quad , \tag{36}$$

where T is expressed in electron volts.

If, on the other hand, the surface potential φ_o is <u>negative</u>, the ion accretion current is enhanced by a factor of $e^{|(T/T_p)\psi_o|}$. With $|\psi_o|$ larger than a few tenths of an electron volt, the enhancement factor $e^{|\psi_o|}$ may be large enough to counteract the effect of the reduction factor $(m_e/m_i)^{1/2}$ so that the ion current may by no means be negligible. In these circumstances, we must include the term

$$J_i = (2\pi/3)\eta_i Z_i n_i v_i R^2 \; e^{-(T/T_p)\psi_o} \tag{37}$$

in calculation of J_p. In writing Equation (37) we have assumed that the ion accretion is also symmetrical about the body. If, however, the speed of the body exceeds the mean thermal speed of the plasma ions by an order of magnitude, the ion accretion current (Equation (37)) is reduced by a factor of 1/2. The corresponding electron accretion current is given by

$$J_e = (2\pi/3)\ \eta_e n_e v_e R^2 e^{(T/T_p)\psi_o} . \tag{38}$$

Therefore, the expression for the plasma accretion current ,
reduces to

$$J_p = J_e - J_i = (2\pi/3) R^2 n_e v_e [\eta_e e^{(T/T_p)\psi_o} - \eta_i (m_e/m_i)^{1/2} e^{-(T/T_p)\psi_o}] . \tag{39}$$

In the calculation of the thermionic escape current we may first remark that the negative surface potential in our problem is only a fraction of a volt. It may also be noted that a negative surface potential, however small, enables all the emitted electrons to escape. Hence, the thermionic escape current is approximately given by

$$J_{esc} = (4\pi RkT)^2 (m/h^3) e^{-\theta} . \tag{40}$$

Finally, in the condition of equilibrium (Equation (35)) Equations (39) and (40) yield

$$\eta_e e^{(T/T_p)\psi_o} - \eta_i (m_e/m_i)^{1/2} e^{-(T/T_p)\psi_o} = 24\pi (m_e/h^3 n_e v_e)(kT)^2 e^{-\theta}$$

$$\simeq 6 \times 10^{29} (T^2/n_e v_e) e^{-\theta} . \tag{41}$$

VI. DISCUSSION

In the preceding sections, we have formulated and analyzed the problem of the screening of the electric potential on a hot spherical object surrounded by an external plasma. It is assumed that 1) the spherical body acquires the electric potential in the processes of the thermionic emission of electrons from the surface of the object and the accretion of the charged particles from the surrounding plasma, and 2) the surface potential and the distribution of the potential and the electron density in the screening cloud are spherically symmetrical about the object. The basic requirement to satisfy these assumptions are that i) the surface of the spherical object is at a uniform temperature and ii) the object is either at rest or it moves with a speed that is small compared to the mean thermal speed of the plasma electrons. These requirements set restrictions on the exact application of the results of the present analysis to actual objects in space. The present analysis, nevertheless, provides, even in such cases where the above-mentioned assumptions do not strictly hold, at least an order of magnitude estimate of this phenomenon in front of the hottest part of the object.

The applications of our analysis may be found in objects entering a planetary atmosphere or those approaching sufficiently close to a hot star. A space vehicle entering the earth's atmosphere encounters stagnation temperatures of the order of 1500°K. All meteoric objects acquire surface temperatures above 1200°K. Ionization in front of the cometary heads and certain cometary tails which is not understood as well, may be attributed in part to the solar heating of the metallic content of these objects. In general, the surface temperatures of the above-mentioned classes of objects are not uniform. Due to the variety of the types of such objects and uncertain available data, we will not make any attempt to apply our analysis to any specific case of the above-mentioned space objects. Instead, we will illustrate our theory by considering a hypothetical spherical object heated to a uniform surface temperature and surrounded by a plasma of electron density $n_e \sim 10^3/cm^3$ at the equilibrium plasma temperature $T_p \sim 1000^{\circ}K$ (0.09 ev). Two values of the work function and five values of the surface temperature, viz.,

$$W_o = 3.0 \text{ and } 3.8 \text{ (electron volts)}$$

and

$$T = 0.04, 0.06, 0.09, 0.13 \text{ and } 0.15 \quad \text{(electron volts)}$$

are considered to illustrate the influence of these parameters on the nature of the electron cloud around a hot object. A common value of $\eta_e = \eta_i = 0.1$ is adopted for the sticking coefficients. Since these surface-plasma parameters appear in a logarithmic term, any departure from this value for the sticking coefficients is not likely to seriously affect our result.

The equilibrium value of the surface potential φ_o is determined by the surface temperature T, the electron density n_e, and temperature T_p of the surrounding plasma. At low values of T, thermionic emission of electrons is small, and hence, the balance of the electron and ion accretion currents from the surrounding plasma establishes a negative potential φ_o on the object's surface. The numerical value of φ_o is always a fraction (≤ 0.05) of a volt because even this small value of φ_o is large enough to increase substantially the ion accretion current and reduce the electron accretion current to off-set the relative effect of the factor $(m_e/m_i)^{1/2}$.

Table 1

SURFACE POTENTIAL OF A METALLIC BODY

$W_o = 3.8$ev	$T_p = 0.09$ev(1044^oK)	$n_e = 10^3$/c.c	$\eta = 0.1$
	T(ev)	φ_o (volt)	
	0.04	-0.1691	
	0.06	-0.1688	
	0.09	+0.0963	
	0.11	+0.4895	
	0.13	+0.8430	
	0.15	+1.1340	

and

ii) $T = 0.11$ ev; $W_o = 3.8$ ev; and $\psi_o = 4.0$

respectively. The variation of potential with distance has the following characteristics:

1) The nature of the profile of the potential distribution curve is independent of the set of the parameters used. The potential falls very rapidly with distance from the object, and reduces to 1/3 of its surface value at a distance of approximately 2.3 and 1.7 cm in Figures 2 and 3 respectively. At a distance of about 8-10 cm the potential acquires an almost zero value and the surface potential of the body is completely shielded by an electron cloud of this dimension.

2) The inclusion or the disregard of the second term inside the parenthesis of Equation (42) does not seem to matter in the calculation of the potential distribution. It is apparently due to the very rapid decrease of potential with distance from the object which reduces this term to a second order of exponential in $\psi - \psi_o$ thereby making it negligible in comparison to the first term.

The electron density in the electron cloud surrounding the body is calculated from Equation (22) by substituting in it the values of the potential distribution obtained from the solution

At high values of T, on the other hand, a positive surface potential of several volts is established by the balance of the thermionic-emission and the electron-accretion currents; the ion-accretion current having been reduced to a negligible value by the joint action of the positive potential and the factor $(m_e/m_i)^{1/2}$. Table 1 lists values of φ_o corresponding to the several values of W_o and T.

Equation (24) can be reduced to a dimensionless differential equation,

$$\frac{1}{X^2}\frac{d}{dX}\left(X^2\frac{d\psi}{dX}\right) = AR^2(KT)^{1/2}e^{-(\psi_o-\psi)}\left[1-(1-X^{-2})^2\right]^{1/2}e^{-(\psi_o-\psi)/(X^2-1)} \qquad (42)$$

where $X = \frac{r}{R}$. The variation of potential with distance from the spherical hot body of 1-cm radius is calculated by solving Equation (42), and the results are illustrated in Figures 2 and 3. The two curves representing the inclusion and the exclusion of the second term inside the parenthesis of Equation (42) for the set of parameters T = 0.09 ev, W_o = 3 ev and ψ_o = 5.43 are shown in Figure 2, while Figure 3 exhibits the profile of the potential distribution, with the inclusion of the second term in Equation (42) in the numerical calculations, for the two sets of parameters:

i) T = 0.13 ev; W_o = 3.8 ev; and ψ_o = 6.5, of Equation (42). The results of this computation are given in Figures 4 and 5. Figure 4, like Figure 2, includes two curves, one of these corresponds to the inclusion of the second term inside the parentheses of Equation (22) while the other disregards this term. The set of parameters used in the computation of these curves have the value $T = 0.09$ ev; $W_o = 3$ ev; $\psi_o = 5.43$. The density distribution curve in Figure 5, as in Figure 3, corresponds to the sets of parameters having values

i) $T = 0.09$ ev; $W_o = 3$ ev; $\psi_o = 5.43$

and

ii) $T = 0.13$ ev; $W_o = 0.38$ ev; $\psi_o = 6.5$

and the numerical calculations are based on the inclusion of the second term in Equation (22). These curves bring out the following features of the variation of the electron density with distance from the object:

1) There is a considerable increase of electron density in the immediate vicinity of the body.

2) The electron density decreases very rapidly with distance from the body.

3) Unlike in the estimates of the potential distribution, the inclusion or exclusion of the second term within the parentheses of Equation (22) in the computations of the electron

density appears to make a substantial difference in these estimates (see Figure 4). The neglect of this term yields a value for the electron density at great distances which is higher than the ambient value. Therefore, it is necessary to consider this term in order to arrive at the correct estimates of the electron density.

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REFERENCES

Chandrasekhar, S. 1939, An Introduction to the Study of Stellar Structure, (Chicago: At the University Press), IV.

Chopra, K.P. 1961, Rev. Modern Phys., 33, pp. 153-189.

Shen, C.S. 1963, J. Atmos. Sci. 21.

CAPTIONS TO FIGURES

Figure 1 - Curves I, II and III are the plots of v_r and v_t in accordance with the Equations (12), (13) and (14) respectively, and define the velocity domains of the thermionic electrons. Electrons in domain A form the escape-group, whereas those in domains B and C describe ballistic orbits with sufficient and insufficient energies respectively to reach position r.

Figure 2 - Plots of $\psi(r)$ against r/R. (W_o = 3ev; T = 0.09ev; and ψ_o = 5.43.) Solid line =

$$\nabla^2\psi = A(kT)^{1/2}[1-\{1-(R/r)^2\}^{1/2}\exp(\psi-\psi_o)F^2/(r^2-R^2)]\exp(\psi-\psi_o).$$

Dotted line = $\nabla^2\psi = A(kT)^{1/2}\exp(\psi-\psi_o)$.

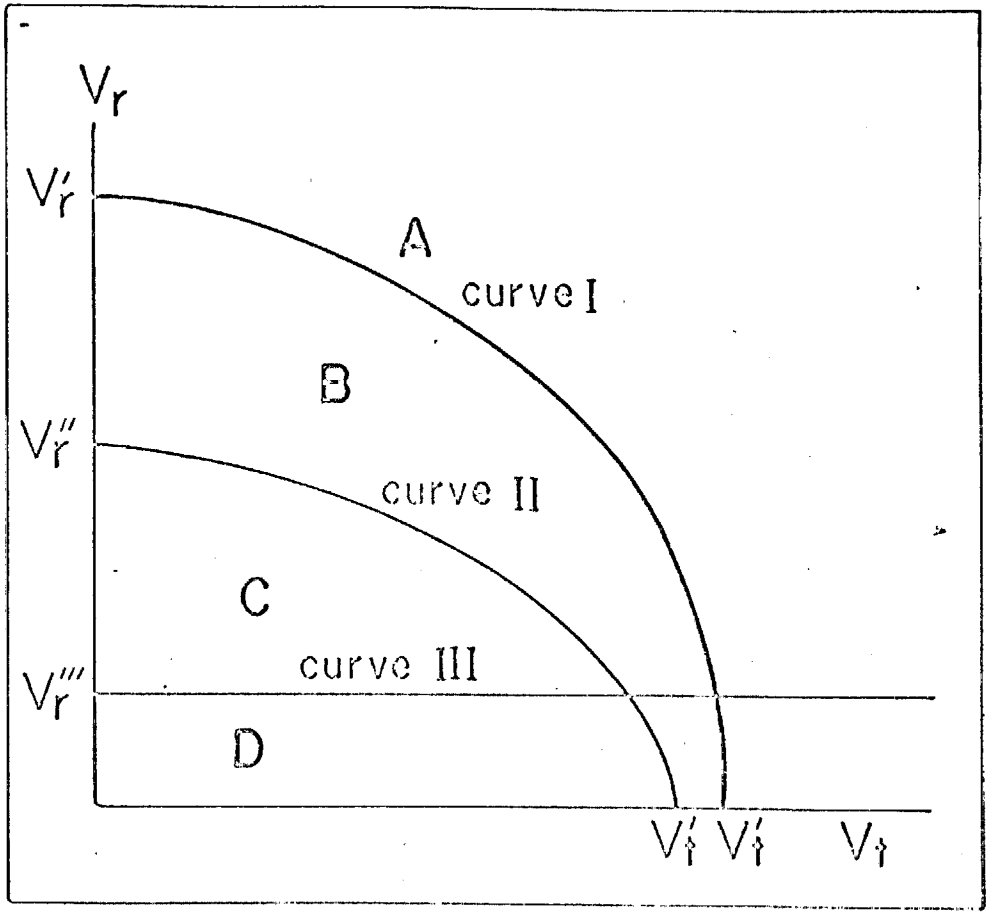
Figure 3 - Solid line = W_o = 3.8ev; T = 0.13ev; ψ_o = 6.50.
Dotted line = W_o = 3.8ev; T = 0.11ev; ψ_o = 4.45.

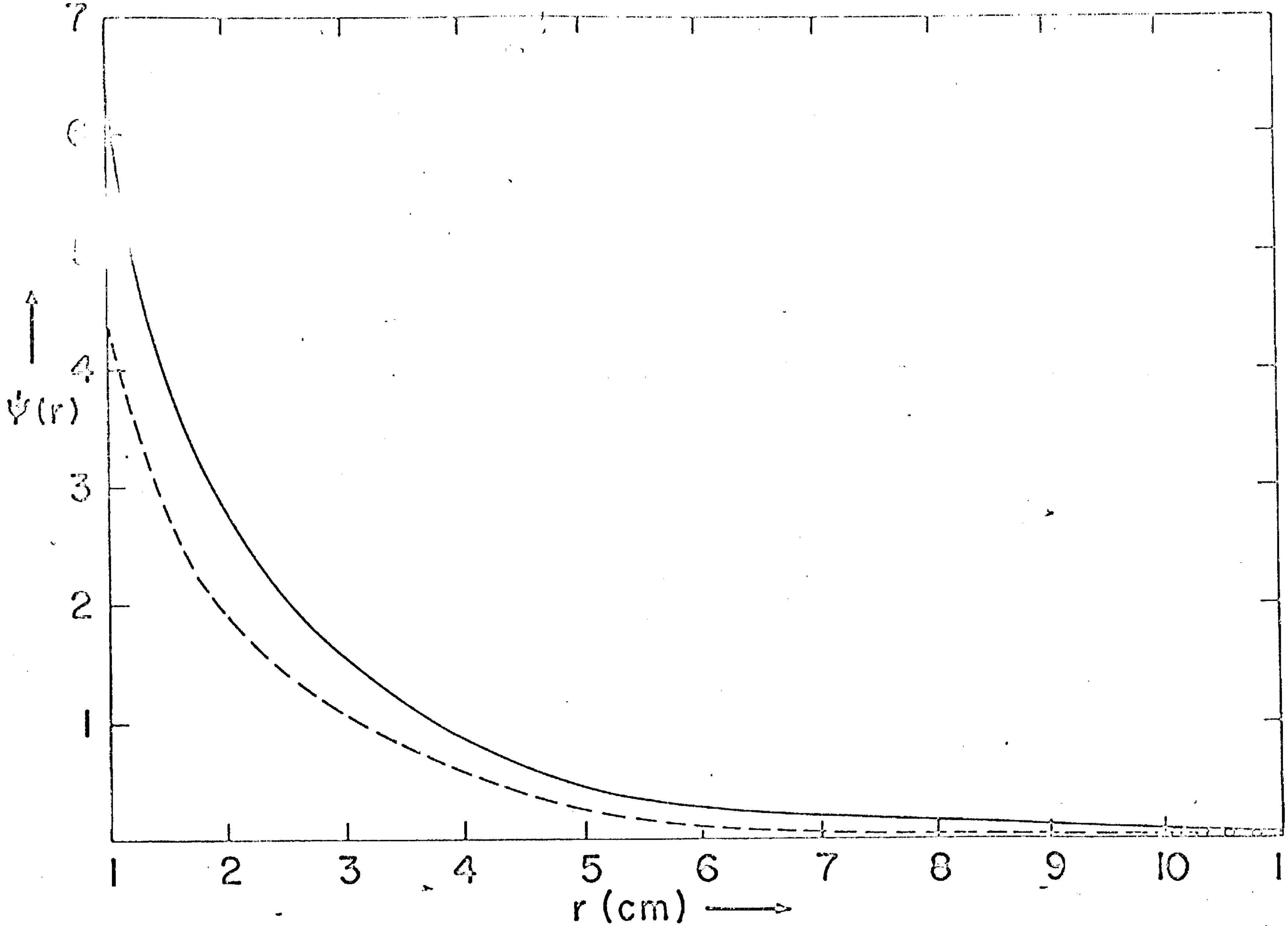
Figure 4 - (W_o = 3ev; T - 0.09ev; and ψ_o = 5.43.) Plots of $\log_n \rho(r)$ against r/R. Solid line =

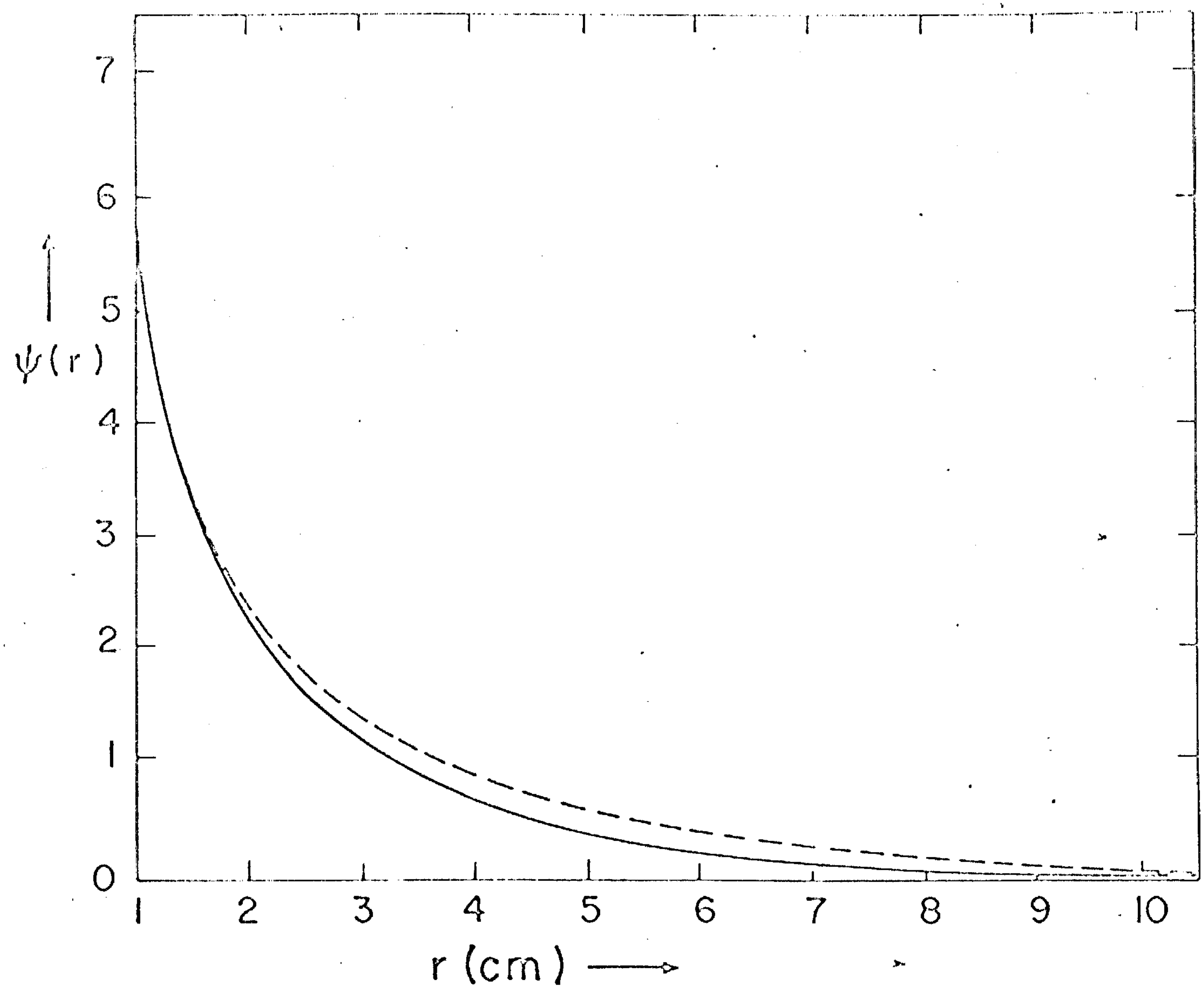
$$\rho = A[1-\{1-(R/r)^2\}^{1/2}\exp(\psi-\psi_o)R^2/(r^2-R^2)]\exp(\psi-\psi_o).$$

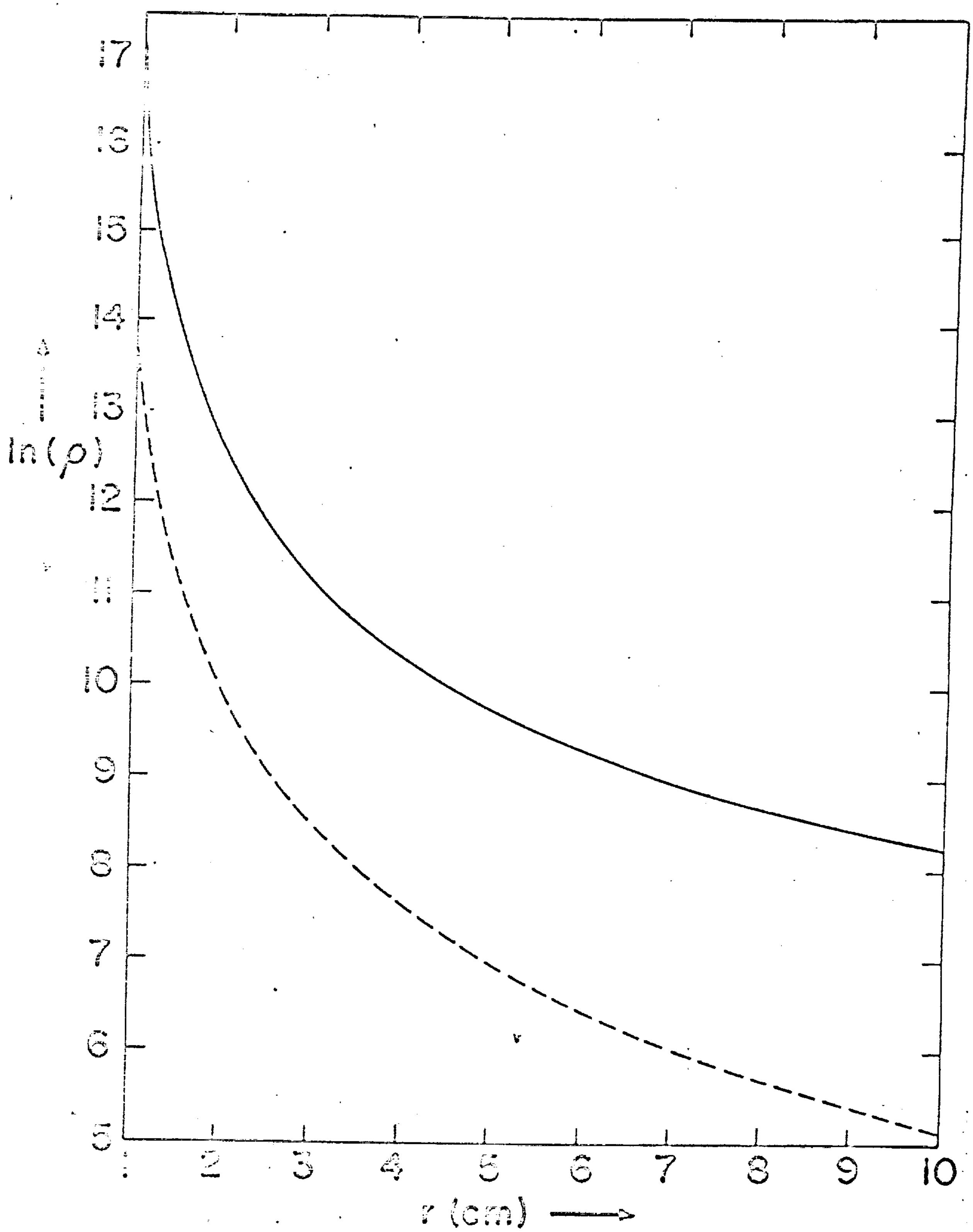
Dotted line - $\rho = A\exp(\psi-\psi_o)$.

Figure 5 - Solid line = W_o = 3.8ev; T = 0.13 3v; and ψ_o = 6.50.
Dotted line = W_o = 3ev; T = 0.09ev; and ψ_o = 5.43.

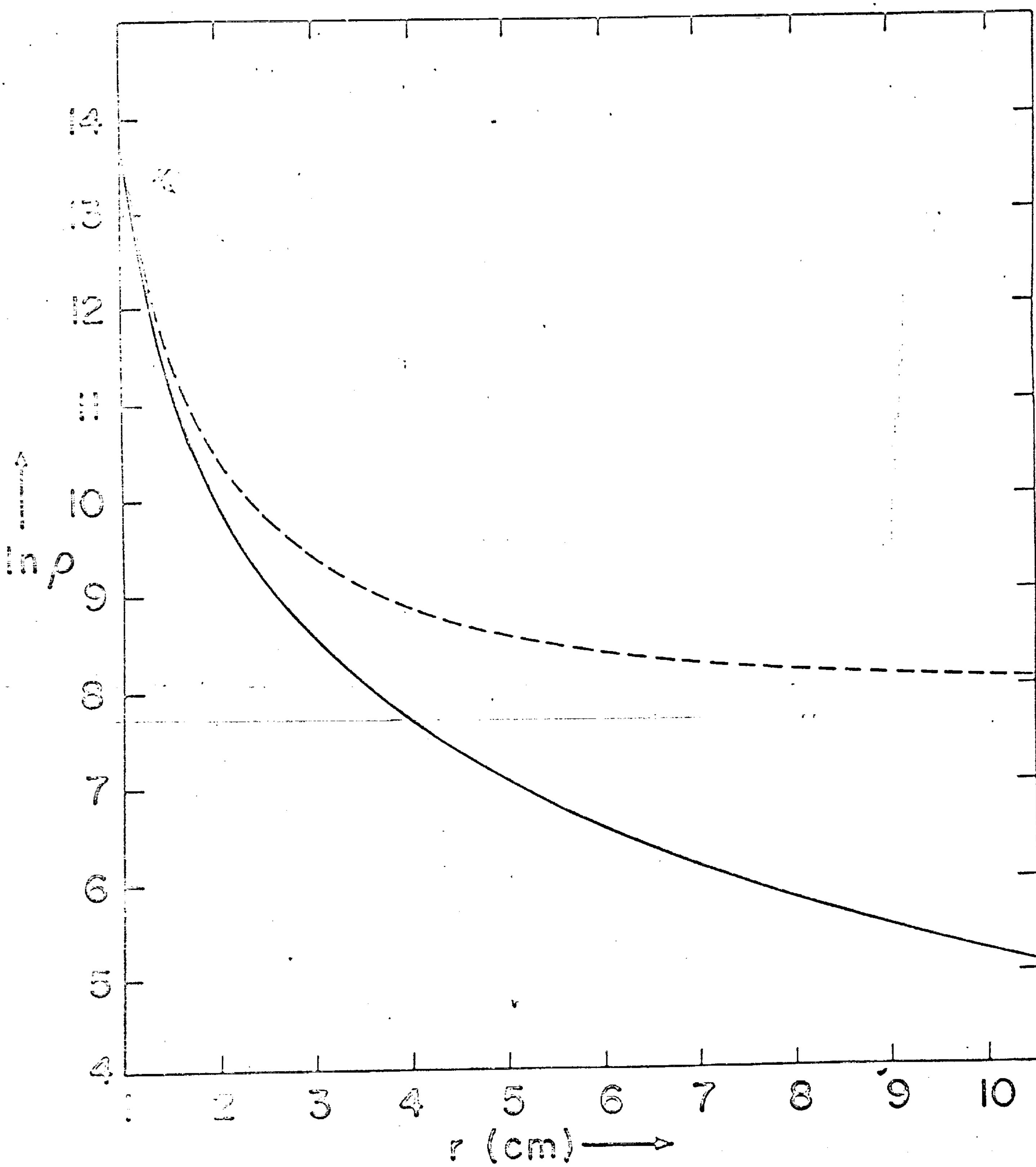

V_r
V_r'
V_r''
V_r'''
A
curve I
B
curve II
C
curve III
D
V_t' V_t'
V_t


7
4
3
2
1
ψ(r)
1
2
3
4
5
6
7
8
9
10
11
r (cm)


7
6
5
4
3
2
1
0
ψ(r)
1
2
3
4
5
6
7
8
9
10
r (cm)



17
16
15
14
13
12
11
10
9
8
7
6
5
ln(ρ)
1
2
3
4
5
6
7
8
9
10
r (cm)



14
13
12
11
10
9
8
7
6
5
4
ln ρ
1
2
3
4
5
6
7
8
9
10
r (cm)